A MODEL FOR SWITCHING IMPULSE LEADER INCEPTION AND BREAKDOWN OF LONG AIR-GAPS

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Abstract - The paper introduces a new mathematical model for continuous leader inception and breakdown of long air gaps under positive switching impulses with critical time-to-crest.

The model deals with rod-, sphere- and conductor-plane gaps. It provides novel analytical expressions for continuous leader inception voltage, height of the final jump and breakdown voltage as well as analytical tools to determine the critical electrode radius for any gap spacing.

The theory is extensively compared with previous experimental results and is tested against several formerly developed empirical formulae, relevant to several discharge parameters, for different electrode forms and over a wide range of gap spacings.

INTRODUCTION

Much work has already gone into the investigation of spark-over characteristics of large air-gaps under switching impulses, particularly rod-plane gaps under positive polarity voltages with critical time-to-crest [1], [2], [3].

The physical picture that emerges from such investigations mainly comprises the start of avalanche activities in the vicinity of the positive rod at a certain threshold field, which may lead to the formation of prebreakdown streamers, normally termed first corona. This may be followed, as the voltage continues to rise, by one or more dark periods during which little activity takes place. Under certain conditions that have largely defied analytical formulation a leader discharge might form at the stem of the streamers. Leader growth may follow which can also be accompanied by one or more dark periods. Under certain circumstances the leader may continuously propagate into the gap, preceded by leader corona. Immediately after leader corona reaches the plane electrode, breakdown of the gap takes place (final jump) with the usual voltage collapse and flow of considerable current.

Practically every stage of the discharge development briefly mentioned above has been extensively studied most notably by members of Les Renardières Group [4], [5].

An important ingredient in those studies was mathematical modelling of different phases of the discharge. These included global models [6], [7], a detailed streamer propagation model [8], a dynamic model for leader propagation [9] and a streamer-plus-leader model [10]. These models contributed significantly to our present day understanding of the discharge mechanism.

Fundamental to the subject of the present paper is a model formulated by Carrara and Thione [11] from which we would particularly underline the following aspects:

- continuous leader inception and propagation, associated with critical time-to-crest was recognized as the most important phase of the breakdown mechanism;
- for sphere-terminated rod-plane gaps and for cylindrical conductor-plane gaps, below a certain critical electrode radius, breakdown voltage remains practically constant independent of the radius.
- for electrodes with radii of curvature greater than the critical radius, first corona and continuous leader inception practically coincide.

For the determination of the leader inception voltage of rod-plane and conductor-plane gaps Carrara and Thione made use of an empirical formula for the 50% sparkover voltage developed at EdF [12] as well as another empirical formula for the height of the final jump due to Lemke [13].

An attempt to physically explain the concept of critical radius in terms of a critical corona intensity was made in [5] using the streamer development model of Gallimberti.

The above introduction leads naturally to the legitimate question: if so much has already been accomplished what is left to justify another paper on the subject?

To the author's knowledge:

- there exists no formula for continuous leader inception voltage, related to gap length and electrode geometry;
- several useful empirical formulae are available for calculation of sparkover voltage of a rod-plane gap, however each is naturally valid within a certain range of gap spacing and bears little or no physical correlation to our present day knowledge of the discharge mechanism;
- critical radius can only be determined from experimental results or empirical formulae;
- there appears to be basic contradiction between a widely used empirical formula for the height of the final jump [13] and the latest sparkover data of very large gaps [14].

The present paper comprises an attempt to address the above aspects of switching impulse discharge phenomena in large air gaps.

THEORY

Fundamental information on switching impulse leader characteristics were provided mainly through contributions of Les Renardières Group [1, 4, 5]. Under a critical positive switching impulse, with crest voltage in the vicinity of U_{50}, the aspects most relevant to the present work include:

- leader propagation is associated with charge injection at a rate q_ℓ = 40-50 μC/m of axial leader length for rod-plane gaps. The linear

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charge density is somewhat higher for spherical electrodes and somewhat lower for cylindrical conductors;

- under critical conditions, the instantaneous applied impulse voltage increases so as to maintain a sensibly constant leader tip potential as the leader continues to penetrate the gap;
- the axial leader velocity is normally in the range 1-2 cm/μs with a frequently quoted value of 1.5 cm/μs;
- with the exception of probable lack of local thermodynamic equilibrium, the leader bears some resemblance to an electric arc, with a voltage gradient that depends on the life time, and which varies from an initial value E_i of approximately 400 kV/m for a newly created leader segment to an ultimate value E_∞ of approximately 50 kV/m. An estimate of the time constant involved is 50 μs [6].

Our main interest in this paper is the continuous leader which, by definition is one that succeeds to penetrate the gap until the final jump and complete breakdown. Contrary to most previous investigations which dealt with discharge phenomena in the vicinity of the positive high voltage electrode, we shall therefore concentrate on the leader conditions as it approaches the final jump.

As the leader propagates into the gap, streamer discharges ahead of the leader (leader corona) will in general develop under the influence of leader space charge being continually injected into the gap at a rate of q_ℓ C/m, the geometric field and streamers own space charge. As will be shown in the following, as the leader approaches the final jump, the field component due to leader space charge becomes most prominent and plays a decisive role in determining the height of the final jump. We shall therefore analyse the electric field due to leader space charge.

Electric Field due to Leader Space Charge

For the purpose of field calculation ahead of the leader tip due to leader space charge, the leader is simulated by a spacial cylindrical distribution of positive charge of longitudinal density q_ℓ situated at an effective radius r_e around the gap axis, having a length ℓ_z along the gap, Fig. 1. To determine the axial electric field E(y) at a point at a distance y from the leader tip, the effect of induced charge on the plane is taken into account through image charges of opposite polarity. The gap spacing is d. The expression of E(y) follows from electrostatics as:

$$E(y) = \frac{q_\ell}{4\pi\varepsilon_o}\int_0^{\ell_z}\left[\frac{x+y}{[r_e^2+(x+y)^2]^{3/2}}+\frac{2(d-\ell_z)+x-y}{\{r_e^2+[2(d-\ell_z)+x-y]^2\}^{3/2}}\right]dx \quad (1)$$

which after performing the integration becomes:

$$E(y) = \frac{q_\ell}{4\pi\varepsilon_o}\left[\frac{1}{\sqrt{r_e^2+y^2}}-\frac{1}{\sqrt{r_e^2+(\ell_z+y)^2}}+\frac{1}{\sqrt{r_e^2+(2d-2\ell_z-y)^2}}-\frac{1}{\sqrt{r_e^2+(2d-\ell_z-y)^2}}\right] \quad (2)$$

Of particular interest is the field E_p, due to the leader space charge, at the plane, for which $y = d-\ell_z$:

$$E_p = \frac{q_\ell}{2\pi\varepsilon_o}\left[\frac{1}{\sqrt{r_e^2+(d-\ell_z)^2}}-\frac{1}{\sqrt{r_e^2+d^2}}\right] \quad (3)$$

which, for $r_e^2 \ll (d-\ell_z)^2$ simplifies to:

$$E_p = \frac{q_\ell}{2\pi\varepsilon_o}\left[\frac{1}{d-\ell_z}-\frac{1}{d}\right] \quad (4)$$

Another quantity that might be of interest is the field at the leader tip due to leader space charge:

$$E_{max} = \frac{q_\ell}{4\pi\varepsilon_o}\left[\frac{1}{r_e}-\frac{1}{\sqrt{r_e^2+\ell_z^2}}+\frac{1}{\sqrt{r_e^2+(2d-2\ell_z)^2}}-\frac{1}{\sqrt{r_e^2+(2d-\ell_z)^2}}\right] \quad (5)$$

which for r_e much smaller than the other distances involved yields the first approximation:

$$E_{max} \simeq \frac{q_\ell}{4\pi\varepsilon_o r_e} \quad (6)$$

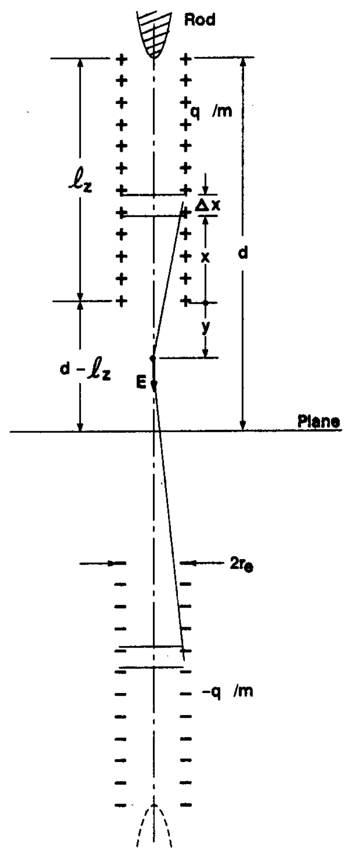

Fig. 1: Schematic Representation of Leader Space Charge.

It is noted that expression (4) provides a simple means to evaluate the electric field at the plane due to the leader space charge as function of the axial leader length for any gap length d.

Before testing expression (4) against actual field probe measurements at the plane, it might be mentioned that earlier efforts [1] using charge simulation techniques failed to reproduce the measured field values due to linear leader charge alone. The main reason as we know now is that earlier injected charge values utilized were incorrectly low. Later digital computations [4] were more successful although the analysis was not pursued any further.

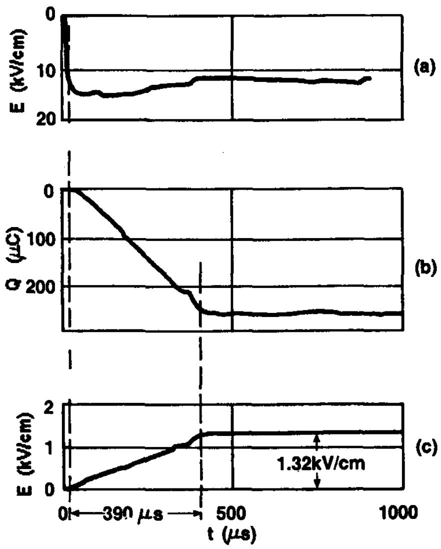

Fig. 2: Simultaneous Records of the Charge (b) Injected into a 10 m Gap (500/10 000 µs impulse) and the Electric Field at the Cone Tip (a) and Earthed Plane (c) from Ref. [4].

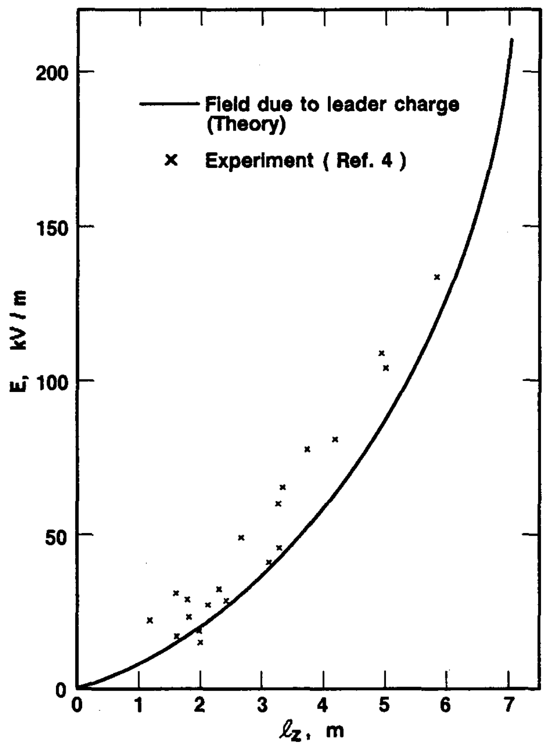

Fig. 3: Field at the Plane due to Leader Linear Space Charge as Function of Axial Leader Length. 10 m Rod-Plane Gap. Critical Switching Impulse. T_{cr} = 500 µs.

The first preliminary test of expression (4) is based on the oscillograms of Fig. 2, produced by Les Renardières Group for injected charge, field at the cone electrode and field at the plane, for a 10 m gap exposed to a 500/10000 µs impulse. The instants of start and end of leader propagation, are identified on the figure, corresponding to an approximate duration of 390 µs. With an average axial velocity of 1.5 cm/µs this travel time corresponds to an axial leader length of 5.85 m. The oscillograms yield q_ℓ of approximately 45 µC/m, which when substituted into expression (4) produces a field at the plane amounting to 1.14 kV/cm. The measured value in Fig. 2 is 1.32 kV/cm. The figure also shows that before the start of leader propagation, at the plane the field was quite small and was severely limited at the rod, indicating an effective discharge shielding effect on the geometrical field for the rod-plane configuration. With these encouraging results we proceed to the more extensive verification of Figure 3. With q_ℓ of 50 µC/m and particularly for longer leader lengths, as the instant of final jump is approached, the results show that the field due to the leader space charge, as calculated above, satisfactorily accounts for the most important part of the field at the plane.

Continuous Leader Inception Voltage of Rod-Plane Gap

The length of a streamer discharge may be determined from the position at which the applied field drops below a certain unknown critical value which is likely to depend on the field configuration [1], while streamer breakdown occurs when the applied voltage amounts to the product of streamer length and mean streamer gradient E_s, which for low breakdown probabilities is approximately 400 kV/m [11].

As the continuous leader reaches the position of final jump, two conditions must therefore be simultaneously fulfilled: the length of the streamers constituting leader corona must be sufficient to reach the plane electrode and the leader tip potential must be adequate to cause streamer breakdown to bridge the remaining gap.

At the position of final jump therefore, we first make the following substitution in (4) above:

$$d - \ell_z = h_f \tag{7}$$

Then the leader tip potential which, under critical conditions, is equal to the continuous leader inception voltage U_{lc} will be given by:

$$U_{lc} = E_s h_f \tag{8}$$

Finally equating the field at the plane due to the leader space charge to the critical field which determines the streamer extent:

$$E_p = E_{cr} \tag{9}$$

Introducing (8) and (9) into (4) we get the following expression for the continuous leader inception voltage:

$$U_{lc} = \frac{E_s}{\frac{2\pi\varepsilon_o E_{cr}}{q_\ell} + \frac{1}{d}} \tag{10}$$

Recognizing that the quantity $q_\ell/2\pi\varepsilon_o$ has the dimension of potential, we substitute

$$\frac{q_\ell}{2\pi\varepsilon_o} = V_o \tag{11}$$

into (10) to get:

$$U_{lc} = \frac{\frac{V_o}{E_{cr}} \cdot E_s}{1 + \frac{V_o}{E_{cr}} \cdot \frac{1}{d}} \tag{12}$$

From (4), (7), (9), (11) we get

$$\frac{1}{h_f} - \frac{1}{d} = \frac{E_{cr}}{V_o} \qquad (13)$$

and

$$h_f = \frac{1}{\frac{E_{cr}}{V_o} + \frac{1}{d}} \qquad (14)$$

In terms of the continuous leader inception voltage U_{1c}, (13) can be written as:

$$\frac{E_s}{U_{1c}} - \frac{1}{d} = \frac{E_{cr}}{V_o} \qquad (15)$$

Equation (15) predicts that the continuous leader inception voltage U_{1c} of a rod-plane, under critical switching impulse conditions, is related to the gap spacing d in such a way as to render the L.H.S. term constant. Equation (15) offers a simple way to test the validity of the present approach. The verification is carried out in Table 1, in which the values of U_{1c} of a rod-plane gap for spacings in the range of 4 to 13.5 m are taken from Carrara-Thione [11]. The last column shows that indeed the L.H.S. of equation (15) is satisfactorily constant with a mean value of 0.257 and a scatter around the mean of ± 5%, which is probably the range of error in the experimental input data.

Table 1
Verification of Equation (15)

d, m	U_{1C}, kV	$\frac{E_s}{U_{1C}}$, m^{-1}	$\frac{E_s}{U_{1C}} - \frac{1}{d}$, m^{-1}
4	800	0.500	0.250
5	880	0.454	0.255
7	1020	0.392	0.249
10	1108	0.361	0.261
13.5	1161	0.345	0.271

Therefore the constant E_{cr}/V_o, for rod-plane gaps, is taken as 0.257. A further experimental check is to use this value obtained for E_{cr}/V_o together with expression (14) to calculate the height of the final jump for a 2 m gap. The so-calculated value amounts to 1.32 m, while the measured value quoted by Carrara-Thione [11] is 1.3 m. Although the absolute value of E_{cr} is not necessary for our further analysis, it may be interesting to get an estimate of that quantity. For q_ℓ between 40 and 50 μC/m the characteristic potential V_o will be in the range 719-899 kV which yields E_{cr} in the range 1.8 - 2.3 kV/cm. Substituting for E_{cr}/V_o by 0.257 in (12) and (14) and for E_s by 400 kV/m in (12) we get the following expressions for the continuous leader inception voltage and height of the final jump of a rod-plane gap under critical switching impulse:

$$U_{1c} = \frac{1556}{1 + \frac{3.89}{d}} \qquad \text{(kV, m)} \qquad (16)$$

$$h_f = \frac{3.89}{1 + \frac{3.89}{d}} \qquad \text{(m)} \qquad (17)$$

Our analytical expression (16) for U_{1c} bears striking resemblance to the well-known EdF empirical formula for U_{50} [12]. A fundamental difference however is that while the EdF formula predicts saturation of U_{50}, formula (16) predicts saturation of the continuous leader inception voltage while, as we shall see later, U_{50} does not saturate.

A further check of (17) is offered through the work of Hutzler [15] who developed an empirical formula for the streamer length ℓ_s ahead of the leader tip for any axial leader length ℓ_z:

$$\ell_s = \frac{4.25}{1 + \frac{3.5}{\ell_z}} \qquad (18)$$

This formula can be used to calculate the height of the final jump since at that position $\ell_s = h_f$ and $\ell_z = d - h_f$. Comparison of h_f as function of ℓ_z between the present paper and the work of Hutzler is shown in Fig. 4. The agreement is satisfactory particularly since for lower leader lengths, Hutzler introduces an additional term in his formula to account for the residual effects of the first corona, which will raise his values of h_f and bring the two curves even closer.

It is interesting to note that both formulae predict *eventual saturation of the height of the final jump*. On the other hand in the empirical formula of Lemke [13], the height of the final jump increases continuously with the logarithm of the gap spacing. If applied to very large gaps this will be incompatible with the results of Pigini et al. [14], which indicate *a linear increase of the breakdown voltage with gap spacing, with a practically constant voltage intercept*.

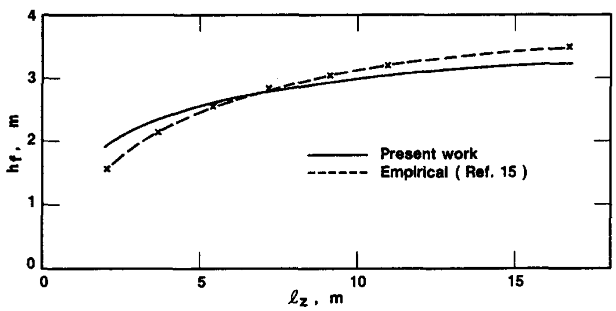

Fig. 4: Height of Final Jump as Function of Axial Leader Length. Rod-Plane Gaps. Critical Switching Impulse.

Leader Voltage Drop

The second component of the minimum breakdown voltage U_B, beside the continuous leader inception voltage U_{1c} is the leader voltage drop ΔU_ℓ.

Due to resemblance between the leader and the electric arc, the leader conductance per unit length G may be assumed [6] to be governed by a dynamic equation due to Hochrainer [16], which for a constant current leader propagation yields

$$G(t) = G_\infty + (G_i - G_\infty)\, e^{-t/\theta} \qquad (19)$$

where t is the life time of the leader section in question, G_i is the initial and G_∞ the ultimate values and θ is a time constant, normally assumed in the range 30-60 μs [6, 9, 15].

Since it is known that the leader, under critical conditions, is propagating with an approximately constant speed v, (19) can be expressed as:

$$G(x) = G_\infty + (G_i - G_\infty)\, e^{-x/x_o} \quad (20)$$

where $x_o = v\theta$, which with $v = 1.5$ cm/μs and $\theta = 50$ μs will be $x_o = 0.75$ m. Here $x = vt$.

Since the leader gradient E_ℓ is related to the conductance per unit length G and leader current i_ℓ by

$$E_\ell = \frac{i_\ell}{G} \quad (21)$$

the leader voltage drop for any axial leader length ℓ_z will be given by:

$$\Delta U_\ell = \int_o^{\ell_z} E_\ell(x)dx = \int_o^{\ell_z} \frac{i_\ell}{G_\infty + (G_i - G_\infty)\, e^{-x/x_o}} dx \quad (22)$$

which after integration yields:

$$\Delta U_\ell = \ell_z E_\infty + x_o E_\infty \ln\left[\frac{E_i}{E_\infty} - \frac{E_i - E_\infty}{E_\infty} e^{-\ell_z/x_o}\right] \quad (23)$$

where E_i and E_∞ are the initial and ultimate values of the leader gradient, normally assumed 400 kV/m and 50 kV/m respectively.

Substituting for the constants in (23) yields:

$$\Delta U_\ell = 50\,\ell_z + 37.5\,\ln[8 - 7\exp(-1.33\,\ell_z)] \quad (\text{kV, m}) \quad (24)$$

For $\ell_z \geq 2$ m this simplifies to:

$$\Delta U_\ell = 50\,\ell_z + 78 \quad (\text{kV, m}) \quad (25)$$

At the final jump, with $\ell_z = d - h_f$, substituting from (17):

$$\ell_z = \frac{d}{1 + \frac{3.89}{d}} \quad (\text{m}) \quad (26)$$

which when substituted in (24), (25) yields:

$$\Delta U_\ell = \frac{50\,d}{1 + \frac{3.89}{d}} + 37.5\,\ln\left[8 - 7\exp\left(\frac{-1.33d}{1 + \frac{3.89}{d}}\right)\right] \quad (\text{kV,m}) \quad (27)$$

and the approximate expression

$$\Delta U_\ell = \frac{50\,d}{1 + \frac{3.89}{d}} + 78 \quad (\text{kV, m}) \quad (28)$$

In Fig. 5, the leader voltage drop expressed by (24) is compared to an empirical formula given by Thione [17].

Continuous Leader Inception of Conductor-Plane Gap

Here we will follow the same procedure used for the rod-plane gap with one exception. In the rod-plane gap, we assumed that due to discharge shielding of the point electrode, the geometric field plays a minor role in determining the field at the plane. For a cylindrical conductor extending considerably beyond the leader, it is more reasonable to assume that the geometric field at the moment of the final jump plays a certain role in the determination of the extent of the streamers constituting the leader corona.

The above arguments lead to the following relationship for the conductor plane gap, which replaces (15):

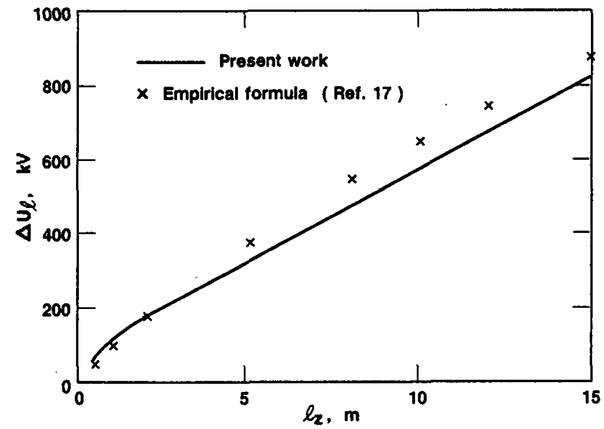

Fig. 5: Leader Voltage Drop as Function of Axial Leader Length.

$$\frac{E_s}{U_{1c}} - \frac{1}{d} + \frac{k_s}{V_o} \cdot \frac{2U_B}{d\,\ln\frac{2d}{r}} = \frac{E_{cr}}{V_o} \quad (29)$$

where k_s is a shielding factor that remains to be determined, and which was assumed to be zero for a rod-plane gap, U_B is the applied voltage at the instant of the final jump ($U_B = U_{1c} + \Delta U_\ell$), and r is the equivalent conductor radius.

It should be noted that both V_o and E_{cr} will in general be different from those of a rod-plane gap. What (29) says in effect is that there must exist a linear relationship between the leader space charge field $V_o(E_s/U_{1c} - 1/d)$ and the geometric field $2U_B/d\,\ln(2d/r)$ at the plane. In order to test this prediction of the theory, we will use values of U_{1c} and U_B from the paper of Carrara-Thione [11] and perform the calculations shown in Table 2, for a conductor radius of 0.05 m as used in ref. [11].

Table 2
Verification of Equation (29)

d, m	U_B, kV	U_{1c}, kV	$\frac{E_s}{U_{1c}} - \frac{1}{d}$, m^{-1}	$\frac{2U_B}{d\,\ln\frac{2d}{r}}$, kV/m
4	1190	1110	0.110	117.2
6	1530	1370	0.125	93.1
8	1770	1540	0.134	76.7
10	1955	1660	0.141	65.3
12	2075	1745	0.145	56.0

First, it is noted that contrary to the rod-plane case, the quantity $E_s/U_{1c} - 1/d$ related to the leader space charge field at the instant of the final jump is not constant, but increases steadily with the gap length. Second, linear regression between $E_s/U_{1c} - 1/d$ and $2U_B/d\,\ln(2d/r)$ according to (29) yielded a correlation coefficient of 0.999 with $E_{cr}/V_o = 0.178$ m^{-1} and $k_s/V_o = 5.767 \times 10^{-4}$ (kV)$^{-1}$. With $V_o = 719$ kV, this corresponds to $E_{cr} = 1.28$ kV/cm and $k_s = 0.4$. After some minor simplifications, (29) yields the following expression for the continuous leader inception voltage of a conductor-plane gap:

$$U_{1c} = \frac{E_s}{\frac{E_{cr}}{V_o} + \frac{1}{d} - \frac{k_s}{V_o} \cdot \frac{2\bar{E}_\ell}{\ln \frac{2d}{r}}} \times \left[1 + \frac{2\frac{k_s}{V_o}(E_s - \bar{E}_\ell)}{\left(\frac{E_{cr}}{V_o} + \frac{1}{d} - \frac{k_s}{V_o} \cdot \frac{2\bar{E}_\ell}{\ln \frac{2d}{r}}\right)^2 d \ln \frac{2d}{r}}\right] \quad (30)$$

Since $\bar{E}_\ell$ depends on ℓ_z, for accurate evaluation $\bar{E}_\ell$ should be determined by iteration.

The quantity in square brackets in (30) is normally around 1.05, so that substituting for the different parameters, a first approximation of U_{1c} will be:

$$U_{1c} \approx \frac{2360}{1 + \frac{5.6}{d} - 6.5 \times 10^{-3} \frac{\bar{E}_\ell}{\ln \frac{2d}{r}}} \quad (\text{kV, m}) \quad (31)$$

and a first approximation for the height of the final jump of conductor-plane gap:

$$h_f \approx \frac{5.9}{1 + \frac{5.6}{d} - 6.5 \times 10^{-3} \frac{\bar{E}_\ell}{\ln \frac{2d}{r}}} \quad (\text{m, kV/m}) (32)$$

APPLICATIONS

Breakdown Characteristics of Rod-Plane Gaps

Following [11] the minimum breakdown voltage of a rod-plane gap can be determined as the sum of U_{1c} and ΔU_ℓ, using (16), (27) and (28) giving:

$$U_B = \frac{1556 + 50d}{1 + \frac{3.89}{d}} + 37.5 \ln \left[8 - 7 \exp\left(\frac{-1.33d}{1 + \frac{3.89}{d}}\right)\right] \quad (33)$$

and for $\ell_z \geq 2$ m i.e. $d \geq 4$ m:

$$U_B = \frac{1556 + 50d}{1 + \frac{3.89}{d}} + 78 \quad (34)$$

The 50% sparkover voltage can be determined in the usual manner as:

$$U_{50} = \frac{U_B}{1 - 3\sigma} \quad (35)$$

where σ is the standard deviation, which for critical switching impulses and rod-plane gaps amounts to approximately 5%. With this value of σ we get the following expression for U_{50} based on (33) and (35):

$$U_{50} = \frac{1830 + 59d}{1 + \frac{3.89}{d}} + 44 \ln \left[8 - 7 \exp\left(\frac{-1.33d}{1 + \frac{3.89}{d}}\right)\right] \quad (\text{kV, m}) \quad (36)$$

and corresponding to (34):

$$U_{50} = \frac{1830 + 59d}{1 + \frac{3.89}{d}} + 92 \quad (\text{kV, m}) \quad (37)$$

The results of computation of U_{1c} and U_{50} for rod-plane gaps in the range 2-27 m are shown in Fig. 6 together with corresponding experimental points. The agreement between theory and experiment for U_{50} is within 2.5% except for the 27 m gap where the theory predicts a value 6% higher than the value reported in ref. [14].

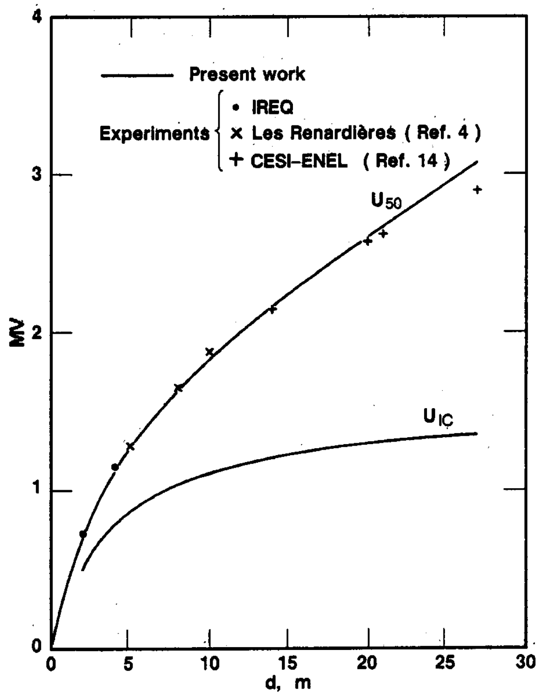

Fig. 6: Continuous Leader Inception Voltage and 50% Sparkover Voltage of Rod-Plane Gaps under Critical Positive Switching Impulses.

Critical Sphere Radius

With the availability of an analytic formula for the continuous leader inception voltage U_{1c}, the determination of the critical sphere radius for any gap spacing can be done by equating U_{1c} to the corona inception voltage U_i.

A formula, based on the streamer theory, for the corona inception field in terms of the mean radius of curvature of the electrode was given in [17]. For a sphere with radius r, m, the formula reads

$$E_i = 2300 \left[1 + \frac{0.224}{r^{0.37}}\right] \quad (\text{kV/m, m}) \quad (38)$$

This field is related to the inception voltage U_i by:

$$E_i = k_g \frac{U_i}{r} \quad (39)$$

where k_g is a geometrical factor related to the sphere support etc. which in effect influences the equivalent radius of the sphere. In Les Renardières experiments [5] k_g can be determined to be in the range 0.8-0.9 for spheres with radii in the range 0.125-0.50 m and gap lengths in the range 3-10 m.

For any gap length the critical radius can then be determined from:

$$\frac{2300}{k_g} r_c \left[1 + \frac{0.224}{r_c^{0.37}}\right] = \frac{1556}{1 + \frac{3.89}{d}} \quad (40)$$

Computation results are shown in Fig. 7, where it is shown that the variation in k_g can account to a great extent for the difference between the experimentally determined critical radii at CESI and Project UHV [18].

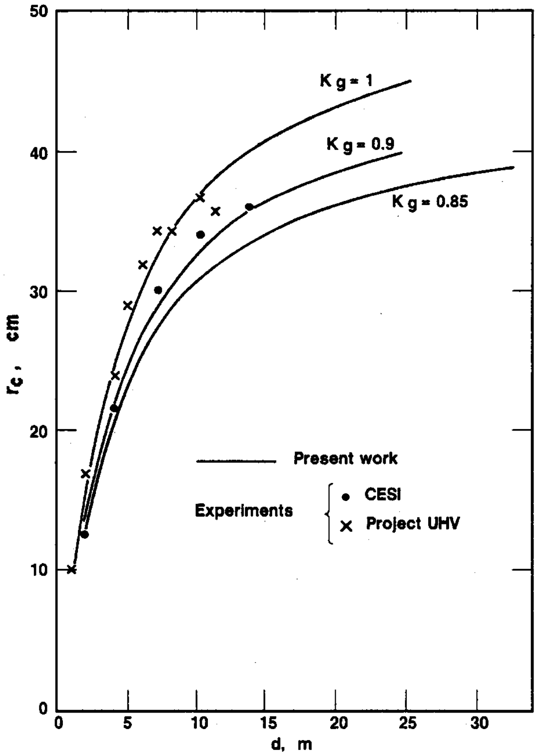

Fig. 7: Critical Sphere-Radius as Function of Gap Spacing. Sphere-Plane Gaps.

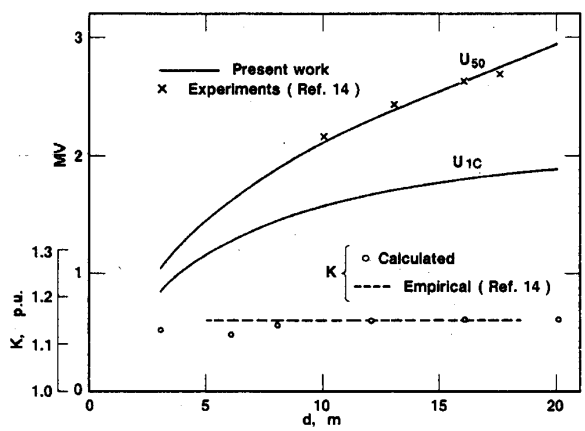

Fig. 8: Positive Switching Impulse Sparkover Characteristics of Conductor-Plane Gaps.

Characteristics of Conductor-Plane Gaps

The same procedure outlined above was followed to calculate the continuous leader inception voltage U_{1c} and U_{50} in the range 3-20 m with one exception: following [14] the standard deviation σ for the conductor-plane gap was taken as 2.5%. The results are shown in Fig. 8 and compared with experiments with r = 0.005 m in both cases. Also indicated is the gap factor computed as the ratio of U_{50} for the conductor-plane and the rod-plane gaps for the same spacing. Had the calculations been made with σ = 5% or had K been determined as the ratio of U_{1c} instead of U_{50}, K would have been in the range 1.22-1.26, instead of the value 1.15 reported in Fig. 8.

To investigate the effect of conductor radius on the sparkover characteristics computations were carried out, for a 12 m gap, with conductor radii in the range 0.005-0.08 m, which as we can see from Fig. 9, is below the critical radius for the gap in question. The results are shown in Table 3. It is remarkable that increasing the conductor radius by 16 fold, but still remaining below the critical radius, did not increase the 50% sparkover voltage by more than 3%.

Table 3
Variation of Sparkover Characteristics with Conductor-Radius for a 12 m Conductor-Plane Gap

r, m	U_{1c}, kV	ΔU_ℓ, kV	U_B, kV	U_{50}, kV
0.005	1676	469	2145	2318
0.020	1706	464	2170	2346
0.040	1727	461	2188	2366
0.060	1741	459	2200	2378
0.080	1753	457	2210	2389

Finally calculations of the critical radius for conductor-plane gaps, with k_g = 1, are shown in Fig. 9.

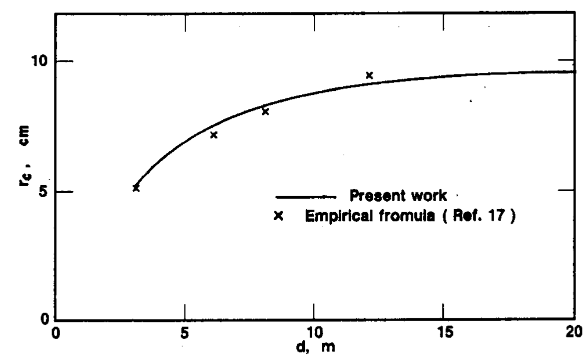

Fig. 9: Critical Radius of Conductor-Plane Gaps as Function of Gap Spacing.

CONCLUSIONS

1. A new model for continuous leader inception and breakdown of long rod-plane gaps under critical positive switching impulses has been developed, based on present knowledge of discharge physics.
2. The model provides novel analytical expressions for continuous leader inception voltage, height of the final jump and breakdown voltage of rod-plane gaps as well as an analytical tool to determine the critical radius for any gap spacing.
3. The present work predicts saturation of the continuous leader inception voltage but no saturation of U_B or U_{50} at large gap spacings.
4. The same basic model has been successfully extended to conductor-plane gaps and has been able, for the first time, to account for the almost constant 50% sparkover voltage for conductor radii below critical.
5. The model covers a very wide range of gap spacings

and successfully accounts for several previously developed empirical formulae relevant to different aspects of the discharge characteristics.

6. The present work explains deviations between previous experimental results related to critical radius that have not been previously accounted for quantitatively.
7. The theory has been subjected to extensive experimental verification and with view of the complex nature of the subject, the agreement is very satisfactory.

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BIOGRAPHY

Farouk A.M. Rizk (Fellow, 82) was born in Egypt on 6 July 1934. He holds a B.Sc.Eng. (1955), M.Sc. (1958) from Cairo University, a Licentiate of technology degree (1960) from the Royal Institute of technology, Stockholm, Sweden and a doctor of technology degree (1963) from Chalmers University of technology, Gothenburg, Sweden.

Dr. Rizk worked as a research engineer with ASEA, Sweden, in the High power Laboratory, Ludvika, (1960-1963) and in the Computer Department, Vasteras, (1963). He worked for the Egyptian Electricity Authority (1964-1971), becoming manager, High Voltage, in 1968. He joined the Institut de recherche d'Hydro-Québec (IREQ) as a senior research scientist in 1972, becoming program manager in 1975, scientific director (1976), director power transmission (1980) and vice-president (1986). Since January 1987 Dr. Rizk holds the title of Fellow Research Scientist at IREQ.

Dr. Rizk was awarded the Egyptian National Prize of Engineering Science for 1971 and decorated by the Order of Worthiness (Third Class) in 1972 and the Order of Science (First Class) in 1973. Dr. Rizk is a registered professional engineer in Quebec and has been, since 1984, international chairman of Technical Committee 28: Insulation Coordination, of the International Electrotechnical Commission (IEC).

Discussion

Prof. M. Khalifa (University of Cairo, Egypt): The author should be congratulated for his excellent contribution. Now, the experimental observations about the breakdown of long air gaps under positive switching surges could be mosaiced by one more scientific formula rather than a number of empirical relations with a limited range each.

The author's model for the leader inception and breakdown in long gaps under atmospheric pressure seems to have valiantly passed numerous checks with experiment. The impression one gets is that the model is correctly based on the proper physical phenomena. The encouraging results given in the paper makes one wonder if only minor modifications are needed to make the model applicable also under switching impulses with fronts shorter, even much shorter than critical. The author's comments would be highly appreciated.

Thinking of the author's successful model fairly accurately predicting breakdown voltages of meter-gaps in atmospheric air, and thinking of Paschen's law, would the model apply equally successfully to gaps of the order of 10 cm at pressures of 10 atmospheres; as expected in GIS? or some adjustment would then be needed?

For the conductor-to-plane gaps, the author cleverly adopted a shielding factor and assigned to it a magnitude of 0.4. Accordingly, the 50% breakdown voltages of a 12 meter gap as calculated came insensitive to the conductor radius so long as it is less than the critical radius. One wonders what would be the suitable magnitude for the shielding factor in case of a gap of only one meter say. The author's comment would be highly appreciated.

Manuscript received January 28, 1988.

A. Pigini (CESI Via Rubattino 54 Milano, Italy): The Author is to be commended for the appreciable work, which led to the formulation of analytical expressions for leader inception voltage and for the breakdown voltage of rod plane and conductor plane geometries holding up to very large clearances.

Most of the approaches and models proposed in the literature, including that of the Author, deal with the various configurations separately: for instance in the paper rod plane and conductor plane geometry is dealt with separately, while some unifying criterion should exhist.

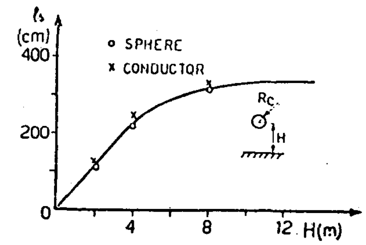

Fig. 1. Streamer length for sphere-plane and conductor plane at leader inception voltage for electrodes having critical radius.

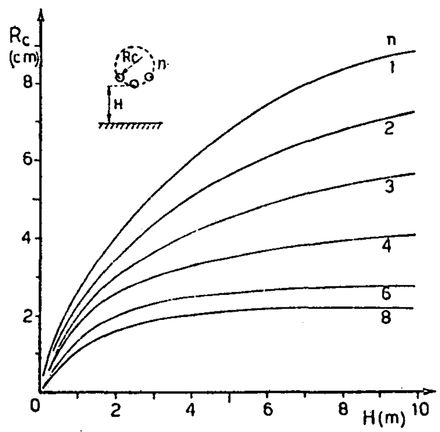

Fig. 2. Computed critical radii for conductor bundles.

An attempt to find a common criterion for electrode-plane geometries was made in [1]. In particular special attention was paid to the leader inception condition of rod plane and conductor plane, with electrodes having critical radii, and it was shown that, for any given electrode height on the ground plane, the streamer length at the leader inception voltage was very close for the two geometries (see Fig. 1). The assumption was made that the above streamer lengths (with related streamer charge) were the minimum ones necessary for streamer to leader transition thus allowing the determination, with a feed back procedure, the critical size of other geometries (e.g. conductor bundles, see Fig. 2). Still the method presents strong limitations due to the apparent dependence of the critical streamer length on the gap clearance, making difficult its application to configurations different from the "electrode-plane" one.

It is obvious that models which could permit to extrapolate the experimental results to complex configurations would be of even greater usefulness than models which substantially help in optimal interpolation of the experimental data for simple configurations.

Do the Author envisage any possibility of extending the approach such to allow the application to other geometries and configurations?

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Manuscript received January 29, 1988.

W. D. Lampe (ASEA-BBC High Voltage Research, Ludvika, Sweden): This paper represents a significant contribution to a deepened understanding of the switching impulse strength of long airgaps. The proposed model finds an impressive support in the available experimental results. The author is to be congratulated.

Not unexpectedly, the critical field strength in the leader and in the streamer before and during the final jump play an important role. The degree of ionization and the size of the discharge channel determine these fields strengths. The question therefore arises if these basic processes for charge creation and the thermodynamics could also be taken into account, which means that the formula can also include the intrinsic dielectric strength of the gas and its fundamental properties.

This might not only provide a sound basis for the density corrections in breakdown strength but also improve the predictions for other gases, in particular those with high electron detachments.

Another question regards the possibility to extrapolate the theory to stepped leaders and then also cover the long gaps bridged by lightning.

Manuscript received February 19, 1988.

Sune Rusck and **Roland Eriksson** (The Royal Institute of Technology, Stockholm, Sweden): The author is to be complimented to an excellent paper, which, starting from the model formulated by Carrara and Thione and using the present knowledge of the physics of long sparks, develops quantitative formulas for the description of the sparkover of long gaps subjected to switching impulses.

The author uses two assumptions for the calculation of the continuous leader inception voltage U_{1c} viz.

$$U_{1c} = E_s h_f \quad (8)$$

$$E_p = E_{cr} \quad (9)$$

These two assumptions together with equation (4) give a formula for the calculation of the final jump h_f

$$h_f = \frac{k}{1 + \frac{k}{d}} \quad (17)$$

where $k = 3{,}89$

In Table A, h_f is calculated as a function of d using eq. (17) with $k = 3{,}89$ and compared with the values given by Carrara and Thione. As can be seen there is a systematic discrepancy between the values calculated by the formula given by Rizk and those quoted by Carrara and Thione. In the same table h_f is also calculated by eq. (17) using $k = 5$ which gives an excellent agreement with the experimental values. This implies that the latter values give a constant critical field strength at the plane, which was one of the postulates of the author.

Table A h_f a function of d

d	2	4	5	7	10	13,5
Rizk	1,32	1,97	2,18	2,50	2,80	3,02
Carrara-Thione	1,30	2,20	2,60	2,95	3,30	3,61
5/(1 + 5/d)	1,43	2,22	2,50	2,92	3,33	3,65

If the value $k = 5$ is accepted, eq. (8) must be changed to

$$U_{1c} = U_0 + E_s h_f$$

A numerical investigation shows that $U_0 = 100$ kV and $E_s = 300$ kV/m which give the formula

$$U_{c1} = \frac{1600 + \frac{500}{d}}{1 + \frac{5}{d}}$$

This formula gives about the same values of U_{1c} as eq. (16). However, with $k = 5$ the length of the final jump and the critical leader length agrees better with the experimental results. The modified leader length should then be introduced into eq. (23) to consider the voltage drop ΔU_1. By suitable choice of constants in eq. (23) it might be possible to improve the agreement between calculated 50% sparkover voltages and experimental results for large spacings as compared in Figure 6.

We would appreciate the author's comments upon our suggestions on a modified formula for U_{1c} based upon one set of reported h_f values. Are there further experimental results avaible by the author or others for comparisons with the proposed modified theoretical approach?

Manuscript received February 19, 1988.

Herman M. Schneider (GE—EPRI High Voltage Transmission Research Center, Lenox, Massachusetts): The author should be commended for making considerable progress in the application of theoretical models to the switching impulse breakdown of large air gaps. It is particularly noteworthy that some of the difficulties in applying concepts such as critical radius have been resolved. However, in extending the present model to the conductor-plane gap, a new complication in the form of a "shielding factor" was introduced to account for the effect of the geometric field near the plane. What quantitative criterion is proposed for deciding on the need for the shielding factor? For example, would a shielding factor be required for a configuration such as conductor-tower, and, if so, what approach is suggested for determining it?

Manuscript received February 19, 1988.

M. Abdel-Salam (Assiut University, Assiut, Egypt): The author should be commended for this interesting paper in which he developed a mathematical model for calculating the continuous leader inception and breakdown voltages of long air gaps under positive switching impulses with critical time-to-crest. The model is based on some assumptions; namely: 1- Axial propagation of the leader. 2- Constant charge injection during leader propagation ($q_l = 45\ \mu C/m$). 3- Constant velocity of leader propagation ($v = 1.5\ cm/\mu s$) 4- Resemblance between the leader and the electric arc with a conductance which varies expotentially with the life time of the leader (time constant $\theta = 50$ us). Subsequently, the voltage gradient within the leader varies from an initial value E_i (=400 kV/m) to an ultimate value E_∞(=50 kV/m). 5- Constant voltage-gradient through the leader corona streamer (E_s = 400 kV/m).

The agreement between the values predicted by the model including the height of the final jump, leader voltage-drop and 50% breakdown voltage and those measured experimentally is excellent in the light of the abovementioned assumptions and the unique values of q_l, v, E_i, E_∞ and E_s. The model dealt with rod-, sphere- and conductor-plane gaps. Was any attempt made to check if the model is applicable for more realistic gap geometries such as tower-window gaps?

The model is restricted to critical conditions, namely switching impulses which lead to the minimum of breakdown voltage. Is there a way to extend the model to take into account the waveform of the applied voltage?

Streak photographs of the luminosity during positive switching impulse breakdown, in a wide variety of electrode geometries, show that the corona clouds (including the leader itself and the streamers at its tip) are conical in shape [1, 2, 3]. Would the author comment on the cylindrical representation of the leader space charge given in Fig. 1 of the paper. The field at the tip of the corona-cloud (E_{cl}) was found [3] to remain approximately constant as the cloud grows, even though the applied voltage increases. When the total length of the leader and streamer exceeds 0.667 of the gap spacing, the developing discharge becomes unstable and E_{cl} increases. The author's comments on these findings in the light of his model are welcomed.

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Manuscript received February 22, 1988.

Gianguido Carrara, CESI, Milano (Italy), EC - The paper represents a noticeable step forward in the so-called "physical approaches" to establish the model under discussion. These approaches are midway between the empirical formulae interpolating test results, and the highly desired, but still not existing, physical models of the whole discharge phenomenon. In my opinion, the success of a physical approach should be based on: a) the number of CONSTANT parameters necessary to calculate the discharge voltages of a given number of insulation configurations; b) whether these parameters are based on physical aspects of the discharge; c) the validity of the model in regions far from the results on which the parameters were evaluated. The Author's model is limited to rod-plane and conductor-plane, but, in this field, meets the three mentioned requirements much better than any similar approach presented up to now. Concerning point a): only eight parameters (q_1, E_s, E_{cr}, G_i, G_{oo}, θ, v, k_s), all CONSTANT, are necessary to calculate the discharge voltages. As concerns point b) all parameters are related to physical elements of the discharge process, except parameter k_s in formula (29). As concerns point c) the parameters were determinad on results obtained up to 13.5 m for rod-plane, and to 12 m for conductor-plane, but give fairly accurate estimates up to 27 m and 17 m respectively, which are the maximum distances for which test results are available. The 6% discrepancy of the result for rod-plane gap of 27 m in Fig. 6 could be reduced adopting a slightly lower value for G_{oo}, which seems more realistic. Furthermore, the Author's model explains differences (Fig. 7) which, in the "critical radius" model, were considered as approximations to be accepted. Now to the questions: can the Author give a more detailed indication on where the parameter E_{cr} is mentioned in ref. [1]? Can the Author express his opinion on what follows? i) My analysis of the model. ii) The physical reasons why k_s is not equal to unity, that is why there is no superposition of the effects. iii) What could be the qualitative changes required to extend the model to phase-to-phase configurations, energized by two voltages of opposite polarity. While expressing my congratulations for the excellent work, I heartily encourage the Author to devote his future efforts to the extension of the model.

Farouk A.M. Rizk: The author would like to thank the discussers for their keen interest, valuable contributions and pertinent questions.

The author would like to bring the following points to the attention of Dr. Abdel-Salam:

1. The cylindrical space charge column representing the leader in the present model should not be confused with the electrically conducting leader channel. As indicated by Hutzler in ref. [9] of the paper, the radius of the space charge column is estimated at 0.5 m or more while the radius of the electrical conduction channel of the leader is of the order of a millimeter.

2. As the leader is almost dark, streak photographs with conical discharge shape refer to streamer discharges ahead of the leader tip (leader corona) and as such are not relevant to our representation of the leader space charge column.

3. As mentioned in the paper, work by Les Renardieres Group [4] showed that a linear space charge along the gap axis is adequate to account for the electric field at the plane. It follows that as long as the radius of the space charge column is much smaller than the length of the final jump, the exact shape of the column is of minor importance to the determination of the field at the plane.

4. As noted by A. Fischer in the discussion of ref. 3 of Abdel-Salam's discussion, the corona cloud model suffers from a fundamental limitation as it assumes the corona-cloud to be a perfect conductor. That is why that model has not been referred to in the present paper.

Professors Rusck and Eriksson suggest to modify formula (16) of the paper, by replacing the constant 3.89 by 5.0, in order to provide better fitting to h_f data in the paper of Carrara-Thione [11]. The author disagrees with such proposal for the following reasons:

1- The discussers are under the false impression that h_f values in ref. [11] were obtained experimentally. In fact, as mentioned in reference [11] and in the present paper, those h_f values for $d > 4$ m were computed from Lemke's model [13]:

$$h_f = 1 + \ln d$$

It is obviously meaningless to try to fit this formula with another formula of the form:

$$h_f = \frac{k}{1 + \frac{k}{d}}$$

2- The suggestion of Rusck and Eriksson necessitates the introduction of an arbitrary formula for U_{1c} and the adoption of an unrealistically low value of E_s = 300 kV/m.
3- The authors of ref. [11] were careful not to use Lemke's model for direct calculation of the leader inception voltage but used it indirectly to determine the leader length, from which the leader voltage drop was deduced, with a relatively minor impact on the accuracy of U_{1c}.
4- The fundamental difference between our formula (16) and that of Lemke for h_f is that the former predicts saturation of the final jump while the latter does not. The paper has already pointed to the incompatibility between Lemke's formula for the final jump at very large gaps and experimental results [14] showing a linear increase of the breakdown voltage with gap length. Such experimental finding, on the other hand, has been fully accounted for by the model of the present paper.

Dr. Pigini presented some interesting results from an earlier attempt to find a common approach to deal with leader inception of electrode-plane geometries based on Gallimberti's model [8]. Pigini admitted however that the method presents strong limitations. It should be mentioned on the other hand, that the approach used in the present paper to deal with rod-plane, sphere-plane and conductor plane configurations remained fundamentally unchanged. The basic criteria for all configurations considered are: the extent of streamers constituting the final jump is determined by a critical value of the applied field and the leader inception voltage must be sufficient to cause streamer breakdown of the final jump. The fact that the relative importance of the different components of the applied field as well as the numerical value of E_{cr} varied between rod-plane and conductor-plane configurations does not by itself constitute a change in approach.

Professor Carrara formulated some objective criteria to evaluate models of physical phenomena and proceeded to show that, within its field of application, the present model meets these criteria much better than any similar approach presented to-date. This is particularly gratifying in view of Prof. Carrara's previous significant contribution to the subject of the paper. The following are the answers to his specific questions:
1. Reference to a critical applied field can be found in the second paragraph of page 116 of ref. [1] which reads: "... there is a minimum value of the charge in the streamer tip below which propagation is not possible. This minimum value (stability charge) depends upon the value of the applied field; at any instant a streamer can grow only if the charge in its tip is higher than the stability charge corresponding to the applied field in the position of its tip".
2. The shielding factor k_s is not equal to unity mainly because in the presence of the leader space charge column, charges of opposite polarity are induced on the high voltage conductor which have the effect of reducing the geometric field component at the plane from its value in the absence of space charge. Further reduction from the theoretical geometrical value for an infinitely long conductor will be caused by the fact that laboratory experiments deal with conductors of finite length and with curved profiles, in order to favour concentration of breakdowns at mid span. Both these effects are included in k_s and can, in principle, be accounted for by numerical computations based on charge simulation.

In answer to Dr. Hermann Schneider, one needs to consider the geometric field component in addition to the leader space charge component at the final jump whenever the electrode configuration plus the leader space charge could not be adequately represented by charge simulation along the gap axis. The factor k_s was introduced in the paper primarily because the objective was to obtain analytical expressions of the leader inception voltage. If numerical solutions are sought, as in the case of more complex configurations, then the geometric field in the presence of the leader space charge will be determined as part of the total applied field. k_s will therefor be obtained implicitly without the need for any special approach to determine it.

In answer to Professor Khalifa's question about k_s, it should be noted that the leader inception voltage is not very sensitive to the value of k_s. For large gaps, an error of 20% in the value of k_s would typically produce an error of 3% in U_{1c}. The one meter gap in particular does not present any difficulty since it breaks down basically through a streamer mechanism. Similarly with impulses much shorter than critical the leader does not have adequate time for significant penetration of the gap.

Dr. Lampe made an interesting suggestion to explore the relationship between the model parameters and fundamental physical properties of the gas. Such effort if successful, may permit the application of the model to other gases and higher presssures as also suggested by Prof. Khalifa.

Finally several discussers proposed to extend the model to more complex structures, phase-phase insulation, impulses other than critical and to lightning. These valuable suggestions will be taken into account in the planning of future work.

Manuscript received June 1, 1988.